Isospin-symmetry-breaking effects in A~70 nuclei within beyond-mean-field approach

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Outline

- *complex* **EXCITED VAMPIR beyond-mean-field model**

- **isospin symmetry in proton-rich A~70 nuclei**

- shape-coexistence and isospin-symmetry-breaking effects on isovector triplets:

Coulomb energy differences (CED)

mirror energy differences (MED)

triplet energy differences (TED)

triplet displacement energy (TDE)

- proton-neutron pairing correlations and competing superallowed Fermi and Gamow-Teller β-decay (^{70}Kr – ^{70}Br , ^{66}Se – ^{66}As)

Characteristic features of proton-rich A~70 nuclei

- *shape transition, shape coexistence, shape mixing*
- *isospin mixing*
- *competing T=0 and T=1 pairing correlations*
- *drastic changes in structure with particle number, spin, excitation energy*

Open problems for theory

- *realistic effective Hamiltonians and beyond-mean-field methods*
- *unitary treatment of structure phenomena at low and intermediate spins and β-decay*

complex VAMPIR model family

- **the model space is defined by a finite dimensional set of spherical single particle states**

- **the effective many-body Hamiltonian is represented as a sum of one- and two-body terms**

- **the basic building blocks are Hartree-Fock-Bogoliubov (HFB) vacua**

- **the HFB transformations are essentially *complex* and allow for proton-neutron, parity and angular momentum mixing being restricted by time-reversal and axial symmetry**

- **the broken symmetries (s=N, Z, I, p) are restored by projection before variation**

* *The models allow to use rather large model spaces and realistic effective interactions*

complex VAMPIR - variational approach to the nuclear many-body problem

with symmetry projection before variation

Effective many-body Hamiltonian

$$\hat{H} = \sum_{i=1}^{M} \varepsilon(i) c_i^\dagger c_i + \frac{1}{4} \sum_{i,k,r,s=1}^{M} v(ikrs) c_i^\dagger c_k^\dagger c_s c_r$$

Hartree-Fock-Bogoliubov transformation

$$\begin{pmatrix} a^\dagger \\ a \end{pmatrix} = F \begin{pmatrix} c^\dagger \\ c \end{pmatrix} = \begin{pmatrix} A^T & B^T \\ B^\dagger & A^\dagger \end{pmatrix} \begin{pmatrix} c^\dagger \\ c \end{pmatrix}$$

$$a_\alpha^+ = \Sigma_{i=1}^{M} (A_{i\alpha} c_i^+ + B_{i\alpha} c_i)$$

$$a_\alpha = \Sigma_{i=1}^{M} (B_{i\alpha}^* c_i^+ + A_{i\alpha}^* c_i)$$

$|i\rangle \equiv |n_i\, l_i\, j_i;\, m_i\, \tau_i\rangle$

Quasi-particle vacuum

$$|F\rangle = \prod_{\alpha=1}^{M'} a_\alpha |0\rangle \quad \text{with} \quad \left\{ \begin{array}{ll} a_\alpha|0\rangle \neq 0 & \text{for } \alpha = 1, \ldots, M' \leq M \\ a_\alpha|0\rangle = 0 & \text{else} \end{array} \right\}$$

Symmetry projection before variation

$$\hat{\Theta}^{s}_{MK} \equiv \hat{P}(I;MK)\hat{Q}(N)\hat{Q}(Z)\hat{p}(\pi)$$

$$\hat{p}(\pi) \equiv \frac{1}{2}\left(1 + \pi\hat{\Pi}\right)$$

$$\hat{Q}(N_\tau) \equiv \frac{1}{2\pi}\int_0^{2\pi} d\phi_\tau \exp\{i\phi_\tau(N_\tau - \hat{N}_\tau)\})$$

$$\hat{P}(I;MK) \equiv \frac{2I+1}{8\pi^2}\int d\Omega D^{I}_{MK}{}^{*}(\Omega)\hat{R}(\Omega)$$

$$|\psi(F^s);sM\rangle = \sum_{K=-I}^{+I} \hat{\Theta}^{s}_{MK}|F^s\rangle f^{s}_{K}$$

$$|\psi(F^s);sM\rangle = \frac{\hat{\Theta}^{s}_{M0}|F^s\rangle}{\sqrt{\langle F^s|\hat{\Theta}^{s}_{00}|F^s\rangle}}$$

Building blocks of the HFB vacuum

$$|F>=\{\prod_{m=1/2}^{m_{max}}(\prod_{\alpha}^{(m)}[u_\alpha+v_\alpha b_\alpha^\dagger b_{\bar\alpha}^\dagger])\}|0>$$

$$b_\alpha^\dagger = \sum_{\tau_i,n_i,l_i,j_i}^{(m_\alpha>0)} D_{i\alpha}^* c_i^\dagger$$

$$[c_{\underline{\mathrm{i}}}^\dagger c_{\underline{\mathrm{k}}}^\dagger]_{TT_z}^{IM} \equiv \sum_{m_i m_k \tau_i \tau_k}(j_i j_k I|m_i m_k M)(\frac{1}{2}\frac{1}{2}T|\tau_i\tau_k T_z)c_i^\dagger c_k^\dagger$$

$$b_\alpha^\dagger b_{\bar\alpha}^\dagger = \sum_{\tau=p,n}\sum_{\underline{\mathrm{i}}\le\underline{\mathrm{k}}}^{(m_\alpha\tau)}[1+\delta(\mathbf{i},\underline{\mathrm{k}})]^{-1}\sum_I(-)^{j_k+l_k-m_\alpha}(j_i j_k I|m_\alpha - m_\alpha 0)$$

$$\times\{[Re(D_{i_\tau\alpha}^* D_{k_\tau\alpha})[1+(-)^{l_i+l_k+I}]+iIm(D_{i_\tau\alpha}^* D_{k_\tau\alpha})[1-(-)^{l_i+l_k+I}]][c_{\underline{\mathrm{i}}}^\dagger c_{\underline{\mathrm{k}}}^\dagger]_{12\tau}^{I0}\}$$

$$+\sum_{\underline{\mathrm{i}}}^{(m_\alpha p)}\sum_{\underline{\mathrm{k}}}^{(m_\alpha n)}\sum_{IT}(1/2 1/2 T|-1/2 1/2 0)(-)^{j_k+l_k-m_\alpha}(j_i j_k I|m_\alpha - m_\alpha 0)$$

$$\times\{[Re(D_{i_p\alpha}^* D_{k_n\alpha})[1+(-)^{l_i+l_k+I}]+iIm(D_{i_p\alpha}^* D_{k_n\alpha})[1-(-)^{l_i+l_k+I}]][c_{\underline{\mathrm{i}}}^\dagger c_{\underline{\mathrm{k}}}^\dagger]_{T0}^{I0}\}$$

Beyond mean field variational procedure

complex VAMPIR

$$E^s[F_1^s] = \frac{\langle F_1^s|\hat{H}\hat{\Theta}_{00}^s|F_1^s\rangle}{\langle F_1^s|\hat{\Theta}_{00}^s|F_1^s\rangle} \qquad |\psi(F_1^s);sM\rangle = \frac{\hat{\Theta}_{M0}^s|F_1^s\rangle}{\sqrt{\langle F_1^s|\hat{\Theta}_{00}^s|F_1^s\rangle}}$$

complex EXCITED VAMPIR

$$|\psi(F_2^s);sM\rangle = \hat{\Theta}_{M0}^s\left\{|F_1^s\rangle\alpha_1^2 + |F_2^s\rangle\alpha_2^2\right\}$$

$$|\psi(F_i^s);sM\rangle = \Sigma_{j=1}^{i}\,|\phi(F_j^s)\rangle\,\alpha_j^i \qquad \text{for} \quad i = 1,...,n-1$$

$$|\phi(F_i^s);sM\rangle = \Theta_{M0}^s|F_i^s\rangle$$

$$|\psi(F_n^s);sM\rangle = \Sigma_{j=1}^{n-1}\,|\phi(F_j^s)\rangle\,\alpha_j^n + |\phi(F_n^s)\rangle\,\alpha_n^n$$

$$\alpha_n^n = \langle\phi^n|[1-\Sigma_{j,l=1}^{n-1}\,|\phi^j\rangle\,(A^{-1})_{jl}\langle\phi^l|]|\phi^n\rangle^{-1/2}$$

$$A_{jl} \equiv \langle\phi^j|\phi^l\rangle \quad i,l = 1,...,n-1$$

$$\alpha_j^n = -\Sigma_{l=1}^{n-1}\,(A^{-1})_{jl}\langle\phi^l|\phi^n\rangle\alpha_n^n$$

$$\hat{S} \equiv \Sigma_{j,l=1}^{n-1} |\phi^j\rangle (A^{-1})_{jl} \langle \phi^l|$$

$$E_1^n \equiv \langle \psi^n | \hat{H} | \psi^n \rangle = \frac{\langle \phi^n | (1-\hat{S}) \hat{H} (1-\hat{S}) | \phi^n \rangle}{\langle \phi^n | (1-\hat{S}) | \phi^n \rangle}$$

$$(H - E^{(n)} N) f^n = 0$$

$$(f^{(n)})^+ N f^{(n)} = 1$$

$$|\Psi_\alpha^{(n)}; sM > = \sum_{i=1}^{n} |\psi_i; sM > f_{i\alpha}^{(n)}, \qquad \alpha = 1, ..., n$$

A~70 **mass** ***region***

^{40}Ca - core

model space for both: protons and neutrons

$1p_{1/2}$ $1p_{3/2}$ $0f_{5/2}$ $0f_{7/2}$ $1d_{5/2}$ $0g_{9/2}$

(charge-symmetric basis + Coulomb contributions to the π-spe from the core)

renormalized G-matrix (OBEP, Bonn A)

- ***pairing properties enhanced by short range Gaussians for:***

  T = 1 pp, np, nn channels

  T = 0, S = 0 and S = 1 channels

- ***onset of deformation influenced by monopole shifts:***

$$\langle 0g_{9/2}\ 0f;\ T=0\ |G|\ 0g_{9/2}\ 0f; T=0\rangle$$

$$\langle 1d_{5/2}\ 1p;\ T=0\ |G|\ 1d_{5/2}\ 1p; T=0\rangle$$

- ***Coulomb interaction between valence protons added***

Isospin-symmetry-breaking effects on

Coulomb Energy Differences

A= 66,70, 82, 86

Exotic case : A = 70

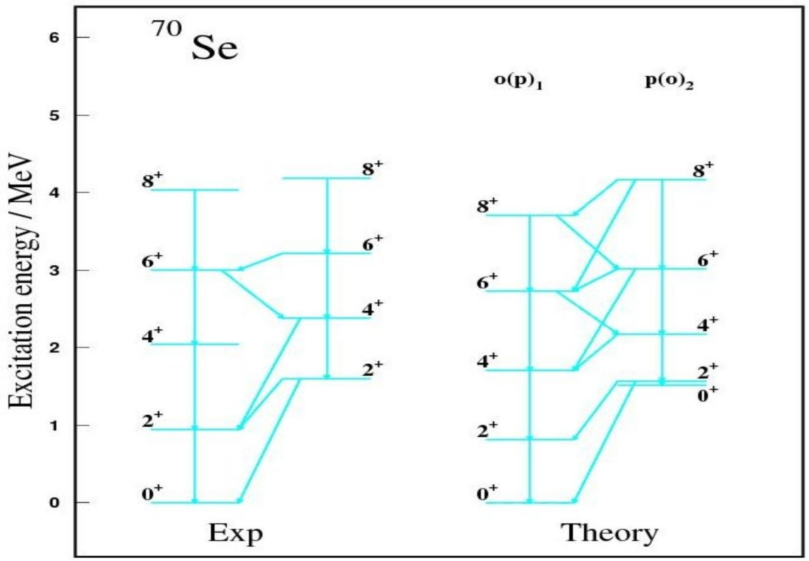
A. M. Hurst et al, Phys. Rev. Lett.98 (2007) 072501
(^{70}Se: No evidence for oblate shapes)

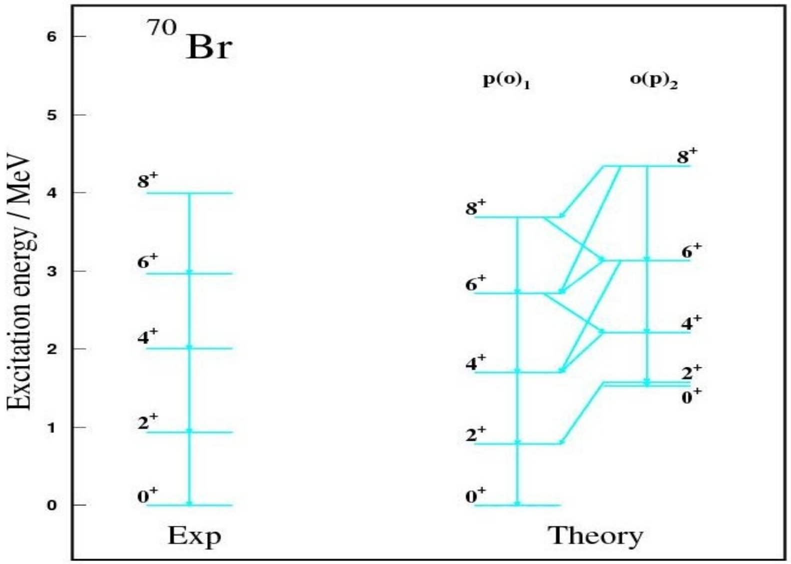
G. de Angelis et al, Eur. Phys. J. A12 (2001) 51 (^{70}Br)

J. Ljungvall et al, Phys. Rev. Lett. 100 (2008) 102502
(^{70}Se: Evidence for oblate shapes)

complex Excited Vampir: *shape mixing and isospin symmetry breaking*

A. Petrovici, J. Phys.G: Nucl. Part. Phys. 37 (2010) 064036





complex Excited Vampir results: oblate-prolate mixing specific for each nucleus varying with increasing spin

Shape mixing manifested in the structure of the wave functions

The amount of mixing for the lowest states in ^{70}Se.

$I[\hbar]$	o-mixing	p-mixing
0_1^+	55%	39%
0_2^+	39%	54%
0_3^+		87%
2_1^+	57%	39%
2_2^+	41%	58%
2_3^+		92%
4_1^+	62%	35%
4_2^+	37%	63%
4_3^+		80(13)%
6_1^+	37%	59%
6_2^+	61%	37%
6_3^+	43%	43%
8_1^+		91%
8_2^+	93%	
8_3^+		84(10)%

The amount of mixing for the lowest states in ^{70}Br.

$I[\hbar]$	o-mixing	p-mixing
0_1^+	35%	62%
0_2^+	59%	34%
0_3^+		88%
2_1^+	41%	57%
2_2^+	58%	40%
2_3^+		94%
4_1^+	41%	56%
4_2^+	57%	41%
4_3^+		94%
6_1^+	20%	76%
6_2^+	79%	20%
6_3^+		44(34)(12)%
8_1^+		89%
8_2^+	96%	
8_3^+		71(11)(11)%

Strong oblate-prolate mixing up to 6^+ : oblate components dominate the yrast states of ^{70}Se, but the yrare states of ^{70}Br

Shape mixing revealed by the spectroscopic quadrupole moments

Spectroscopic Q_2^{sp} (in efm^2) of the lowest three states of spin I of ^{70}Se (effective charges $e_p = 1.2$, $e_n = 0.2$).

$I[\hbar]$	I_1	I_2	I_3
2^+	4.5	-7.	-43.7
4^+	11.5	-16.8	-54.4
6^+	-17.5	9.5	-54.2
8^+	-64.	52.1	-60.

Spectroscopic Q_2^{sp} (in efm^2) of the lowest three states of spin I of ^{70}Br (effective charges $e_p = 1.2$, $e_n = 0.2$).

$I[\hbar]$	I_1	I_2	I_3
2^+	-6.4	4.6	-44.6
4^+	-9.8	5.2	-60.8
6^+	-39.7	33.7	-62.2
8^+	-65.5	59.	-71.4

Precise quadrupole moments for low spin states could clarify the open problem

Shape mixing revealed by the B(E2;ΔI = 2) strengths

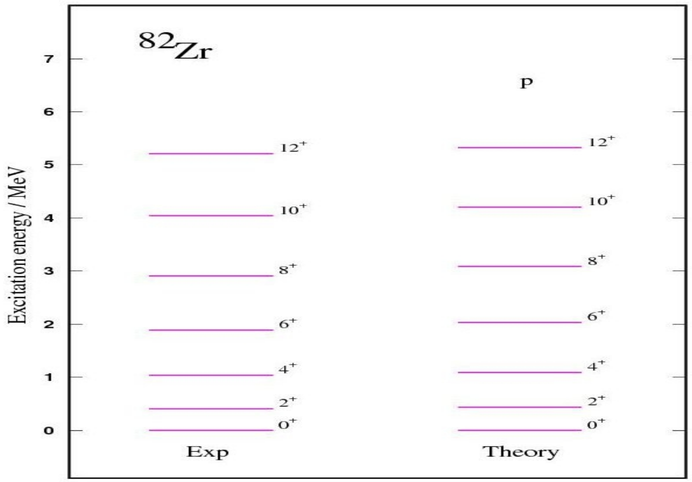
$B(E2; I \rightarrow I-2)$ values (in e^2fm^4) for the lowest two bands of ^{70}Se (EXVAM). Strengths for secondary branches are given in parentheses (effective charges $e_p = 1.2$, $e_n = 0.2$) .

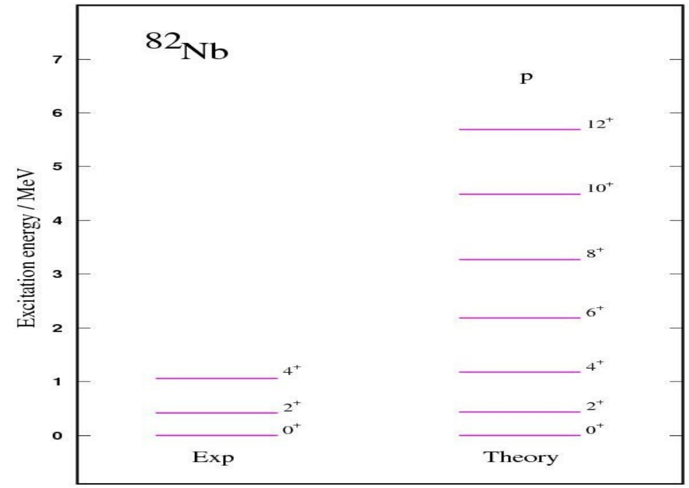
$I[\hbar]$	EXVAM $o(p)_1$	EXVAM $p(o)_2$	Exp.	(HFB-based-config.mix.) (Girod et al.)
2^+	492	501 (5)	342 ± 19	549
4^+	713	761	370 ± 24	955
6^+	779 (62)	792 (33)	530 ± 96	1404
8^+	717 (193)	666 (150)		

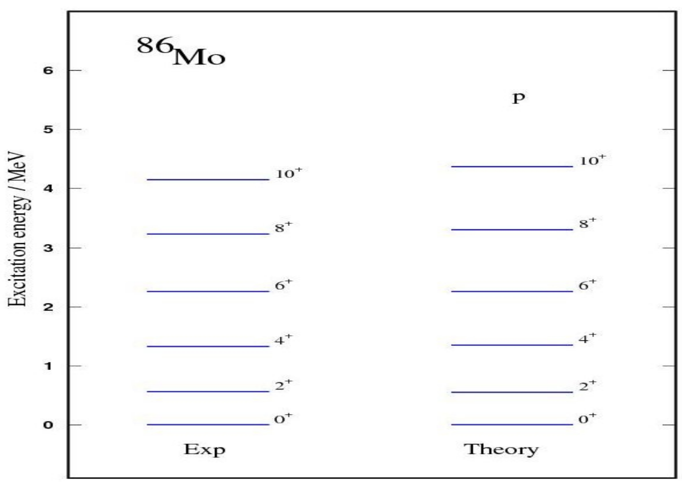
$B(E2; I \rightarrow I-2)$ values (in e^2fm^4) for the lowest two bands of ^{70}Br (EXVAM). Strengths for secondary branches are given in parentheses (effective charges $e_p = 1.2$, $e_n = 0.2$).

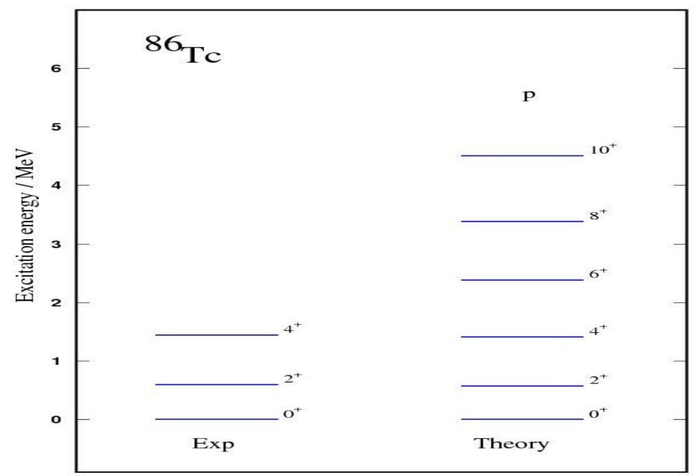
$I[\hbar]$	$p(o)_1$	$o(p)_2$
2^+	541	516
4^+	775	756
6^+	820 (60)	777 (44)
8^+	771 (81)	754 (84)

A = 82, 86 *analogs*









one prolate deformed configuration dominates (>90%) the structure of the yrast states

A. Petrovici et al., Phys. Rev. C78 (2008) 064311

New exotic case: A = 66 ?

$^{66}As - ^{66}Ge$ → *G. de Angelis, A. Petrovici et al., Phys. Rev. C85 (2012) 034320*
P. Ruotsalainen et al., Phys. Rev. C88 (2013) 024320

complex Excited Vampir results: *different shape mixing changing with spin*

The amount of mixing of the lowest states in ^{66}Ge.

$I[\hbar]$	o-mixing	p-mixing ($\mathbf{p}_s$)
0_1^+	18(2)%	77(**2**)(1)%
0_2^+	4%	**82**(10)(4)%
2_1^+	38%	59(2)%
2_2^+	57%	37(6)%
4_1^+	32%	65(1)%
4_2^+	63%	33(3)%
6_1^+	9%	90(1)%
6_2^+	82%	9(5)(3)%

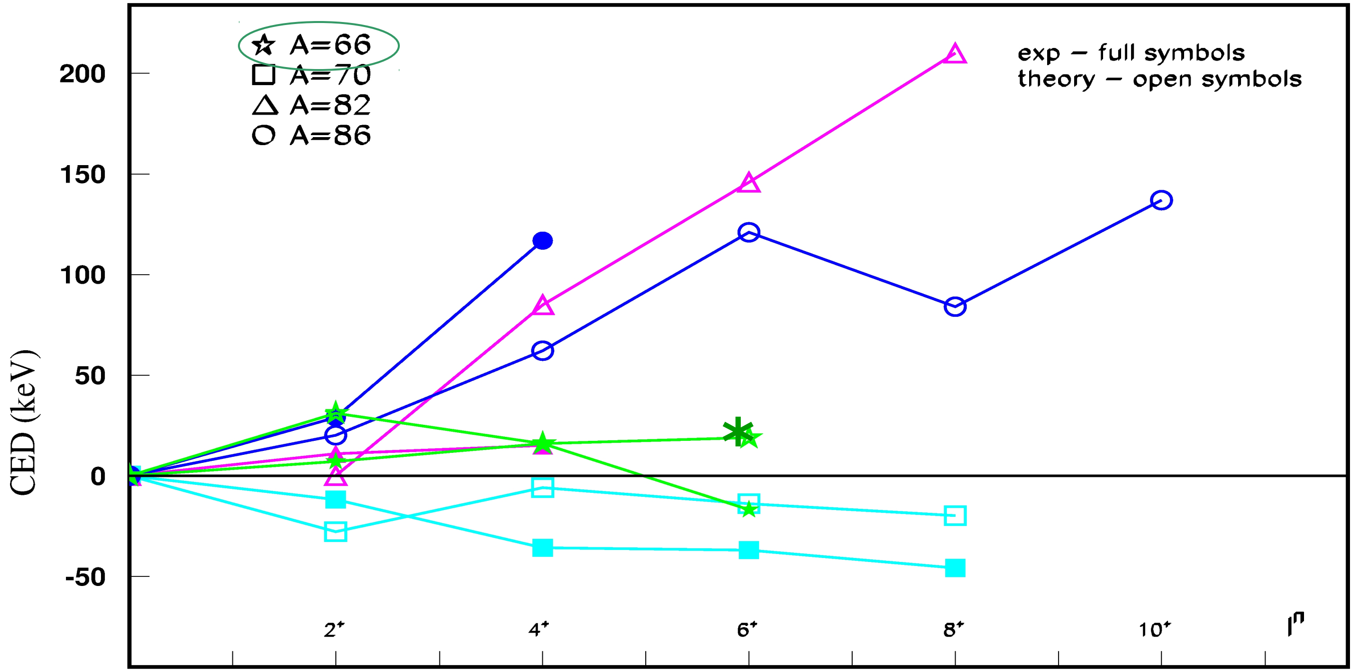
The amount of mixing of the lowest states in ^{66}As.

$I[\hbar]$	o-mixing	p-mixing ($\mathbf{p}_s$)
0_1^+	15(1)%	80(**2**)(2)%
0_2^+	2(2)%	**76**(12)(7)(1)%
2_1^+	29%	68(2)%
2_2^+	64%	31(3)(1)(1)%
4_1^+	18%	80(1)%
4_2^+	76%	18(5)(1)%
6_1^+	4%	95(1)%
6_2^+	81%	14(4)%

Significant oblate-prolate mixing up to 6^+: prolate components dominate the yrast states of ^{66}Ge and ^{66}As

(I) Coulomb energy differences (CED)

$$CED_{J,T} = E^*_{J,T,Tz=0} - E^*_{J,T,Tz=+1}$$



A. Petrovici, J. Phys.G: Nucl. Part. Phys 37 (2010) 064036

* *G. de Angelis, A. Petrovici et al., Phys. Rev. C85 (2012) 034320*

* *P. Routsalainen et al., Phys. Rev. C88 (2013)024320*

(II) *Mirror Energy Differences*

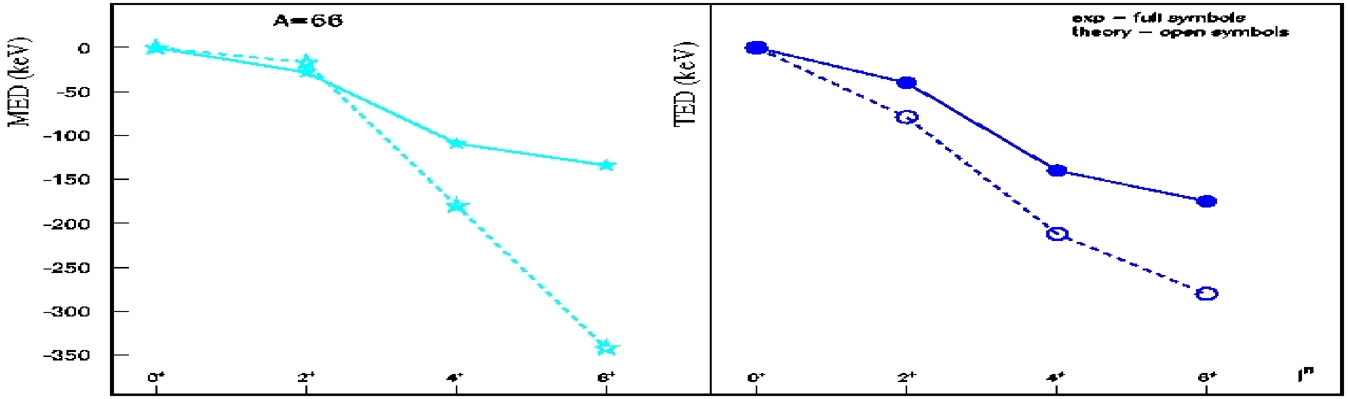
(III) Triplet Energy Differences

$$MED_{J,T} = E^*_{J,T,Tz=-1} - E^*_{J,T,Tz=+1}$$

$$TED_{J,T} = E^*_{J,T,Tz=-1} + E^*_{J,T,Tz=+1} - 2E^*_{J,T,Tz=0}$$

A=66 isobaric triplet (^{66}Se - ^{66}As - ^{66}Ge)

(^{66}Se, P.Ruotsalainen et al. Phys. Rev. C88 (2013) 041308(R))



Competing β-decays within the A=70 and A=66 isovector triplets and proton-neutron pairing correlations

^{70}Kr → ^{70}Br → ^{70}Se : superallowed Fermi β-decay *(A. Petrovici et al, Nucl.Phys. A747 (2005) 44)*

^{70}Kr → ^{70}Br : competing superallowed Fermi and Gamow-Teller β-decay

(T=0 n-p pairing ???, Iachello, Padova, 1994) *(accepted proposals at RIKEN, 2014)*

^{70}Kr - ^{70}Br - ^{70}Se : MED, TED *(accepted proposal at Jyvaskyla, 2014)*

Pair structure analysis

pair number operator

$$\rho_{(M)}^{JTT_z\pi} \equiv \frac{1}{2} \sum_{n_i l_i j_i n_k l_k j_k} \delta((-)^{l_i+l_k}, \pi)(-)^{j_i+j_k-M}(-)^{1-T_z}$$
$$\times \sum_{m_i m_k \tau_i \tau_k} \langle j_i m_i j_k m_k | J M \rangle \langle \frac{1}{2}\tau_i \frac{1}{2}\tau_k | T T_z \rangle c^\dagger_{n_i l_i j_i m_i \tau_i} c^\dagger_{n_k l_k j_k m_k \tau_k}$$
$$\times \sum_{m_r m_s} \langle j_k - m_r j_i - m_s | J - M \rangle \langle \frac{1}{2} - \tau_k \frac{1}{2} - \tau_i | T - T_z \rangle c_{n_k l_k j_k m_r \tau_k} c_{n_i l_i j_i m_s \tau_i}$$

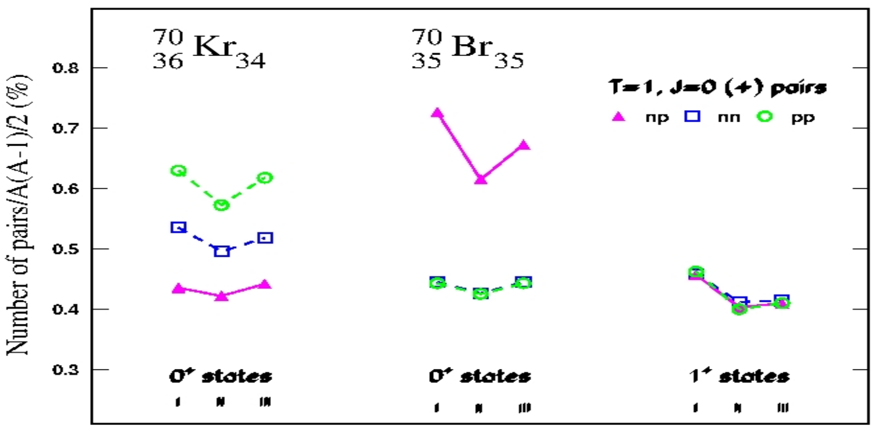
A=70 : complex EXCITED VAMPIR results

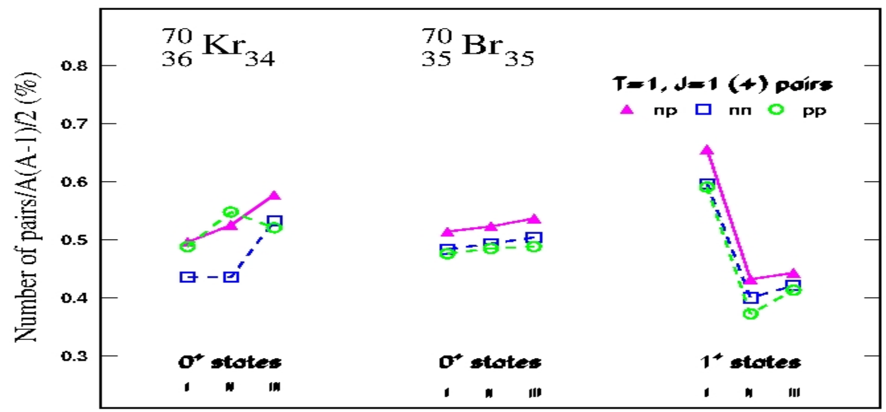
^{70}Kr $0^+_I \rightarrow$ *49% oblate / 51% prolate*
$0^+_{II} \rightarrow$ *44% oblate / 56% prolate*
$0^+_{III} \rightarrow$ *14% oblate / 86% prolate*

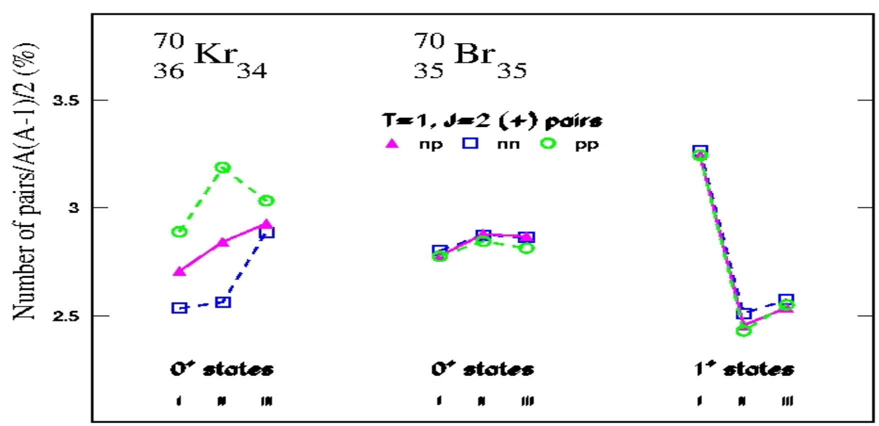
^{70}Br - lowest 1^+ states (1.9 MeV, 2.6 MeV, 2.9 MeV)

one dominant EXVAM configuration

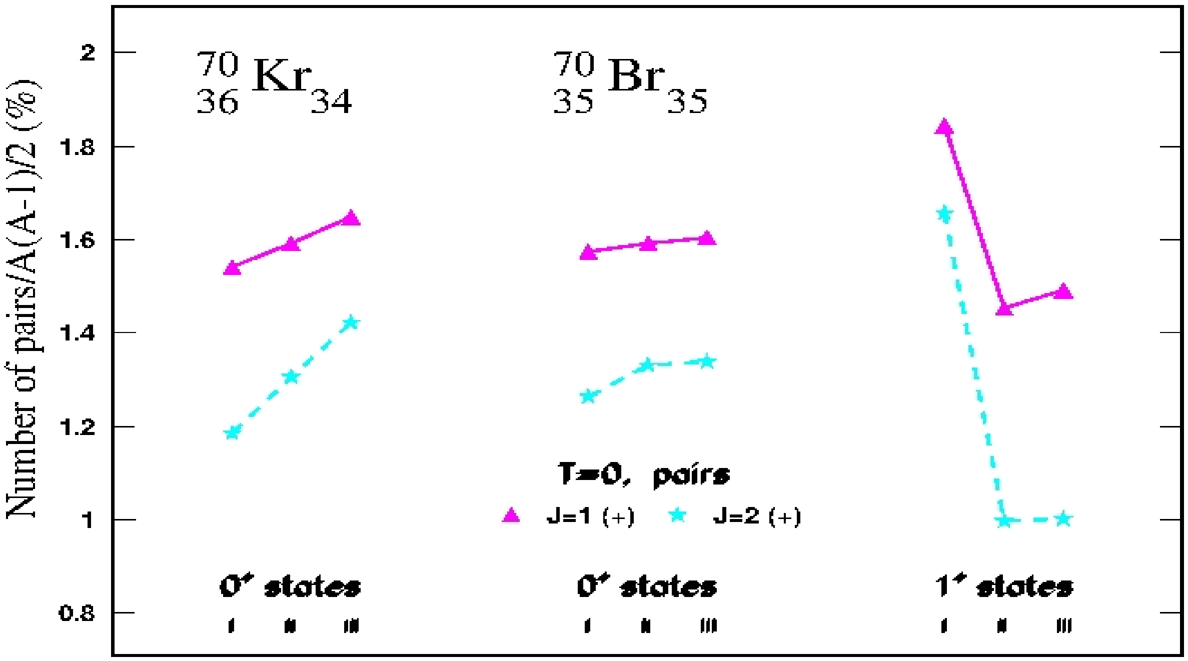
$1^+_I \rightarrow$ *oblate* $1^+_{II} \rightarrow$ *prolate* $1^+_{III} \rightarrow$ *prolate*







B(GT) : $0^+_{gs} \rightarrow 1^+_I$ (**negligible**) / *1^+_{II}* (**0.24 $g^2_A/4\pi$**) / *1^+_{III}* (**0.16 $g^2_A/4\pi$**)



No enhancement of proton-neutron T=0 pairing correlations for Gamow-Teller contributing low-lying 1^+ states

(A. Petrovici, Rom. Journ. Phys.58 (2013) 1120)

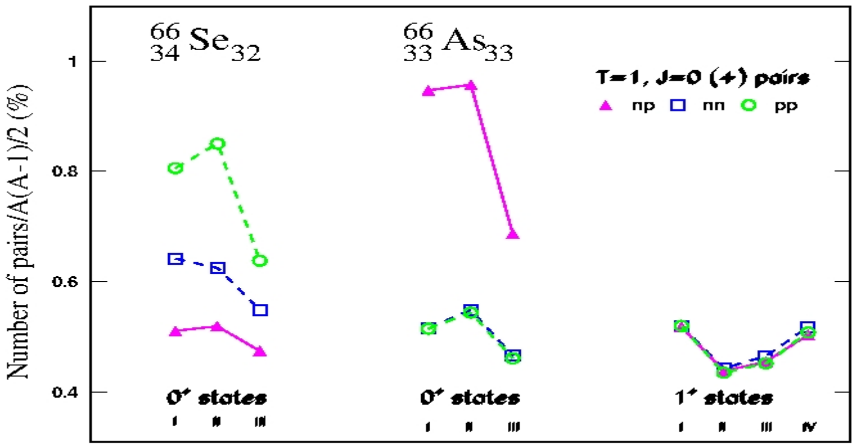
A=66 : complex EXCITED VAMPIR results

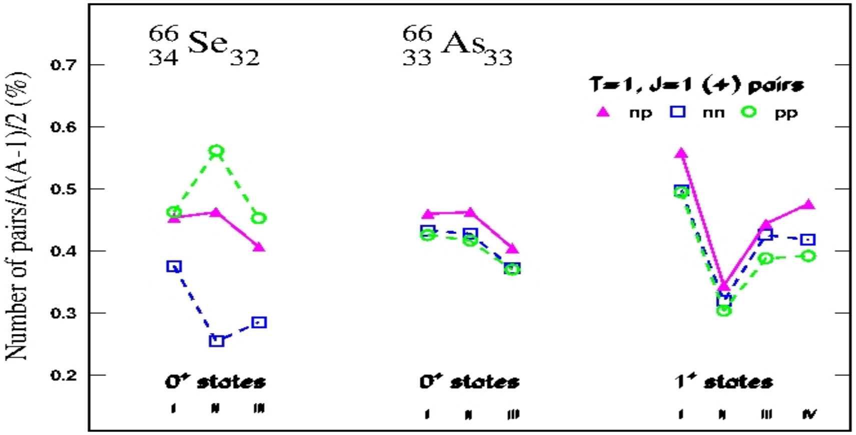
^{66}Se

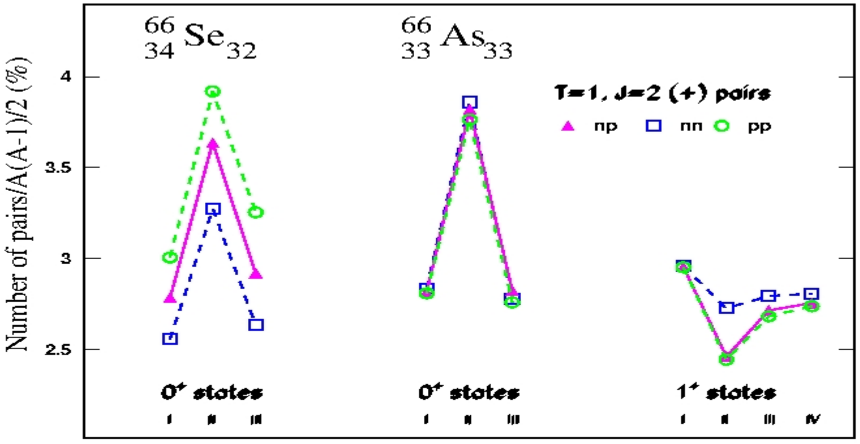
$0^+_I \rightarrow$ 15% oblate / 85% prolate
$0^+_{II} \rightarrow$ 3% oblate / 97% prolate
$0^+_{III} \rightarrow$ 34% oblate / 66% prolate

^{66}As - lowest 1+ states:

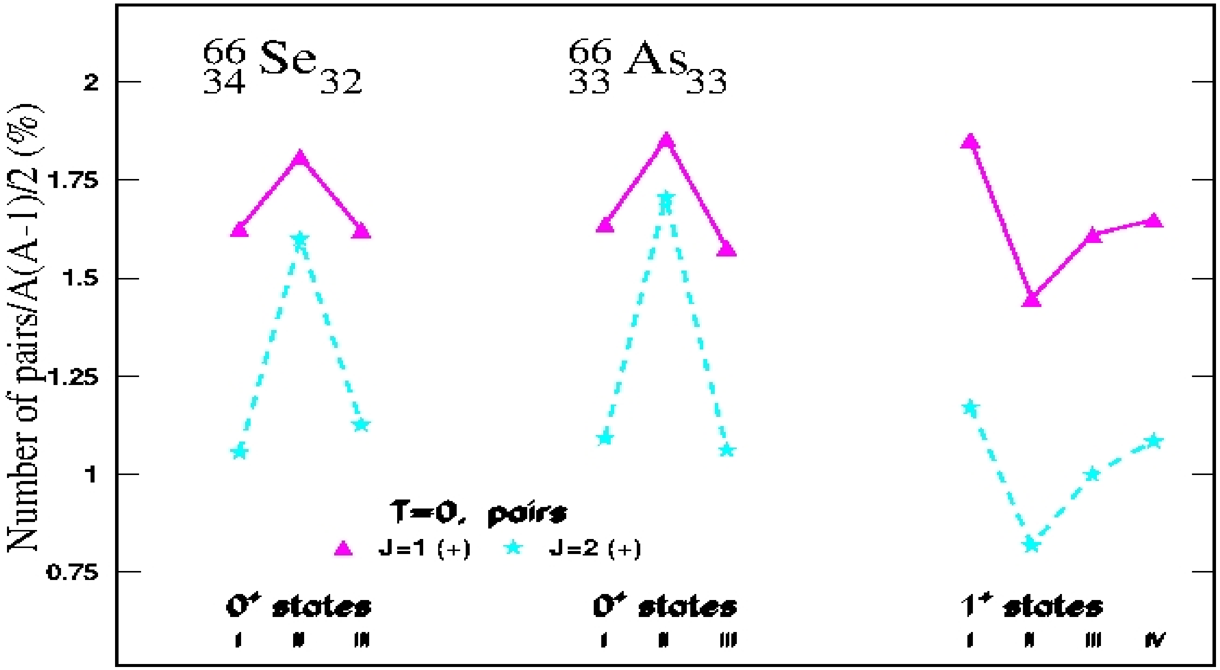
$1^+_I \rightarrow$ oblate configuration
$1^+_{II, III, IV} \rightarrow$ prolate mixing







***B(GT): $0^{+}_{gs} \rightarrow 1^{+}_{I}$* (negligible) / 1^{+}_{II} (0.08 $g^2_A/4\pi$) / 1^{+}_{III} (0.16 $g^2_A/4\pi$)/ 1^{+}_{IV} (0.37 $g^2_A/4\pi$)**



No enhancement of proton-neutron T=0 pairing correlations for Gamow-Teller contributing low-lying 1⁺ states

(preliminary results)

Summary and outlook

complex **EXCITED VAMPIR model** self-consistently describes

- **the interplay between shape-coexistence and isospin-mixing effects on :** ***CED, MED, TED*** **and** ***TDE*** **in A~70 isovector triplets**
- **the possible competition between superallowed Fermi and Gamow-Teller** $\boldsymbol{\beta}$**-decay in A~70 triplets**

  → *does not support the proposed scenario of enhancement of GT branch to the yrast* 1^+ *state in the N=Z odd-odd daughter nucleus as fingerprint of neutron-proton T=0 pairing condensate*

The investigations on the structure and dynamics of medium mass isovector triplets are currently extended taking into account charge dependence in the strong force