Triple shape coexistence and β decay of ^{96}Y to ^{96}Zr

A. PETROVICI

Horia Hulubei National Institute for Physics and Nuclear Engineering, Bucharest, Romania

Outline

- *complex* **EXCITED VAMPIR beyond-mean-field model**
- **neutron-rich A~100 nuclei with N~58**

in collaboration with

- triple shape coexistence and shape evolution in ^{96}Sr and ^{98}Zr.		*(2012)*
- shape coexistence and Gamow-Teller β^- decay of ^{102,104}Tc	*(B. Rubio – Valencia Univ.)*	*(2013)*
- oblate-prolate coexistence and shape evolution in ^{94}Kr	*(G. de France – Ganil)*	*(2017)*
- particular behavior of N=56 ^{100}Ru	*(C. Petrache – IN2P3– Orsay)*	*(2017)*
- shape coexistence and new isomers in ^{96}Y	*(S. Leoni – Milano Univ.)*	*(2017)*

**triple shape coexistence and β^- decay of ^{96}Y (N=57) to ^{96}Zr (N=56)*

- *first-forbidden β^- decay of the 0^- ground state of ^{96}Y*
- *Gamow-Teller β^- decay of the 8^+ isomer in ^{96}Y*
- *exotic decays of the 0^+ daughter states in ^{96}Zr*

Characteristic features of neutron-rich A~100 nuclei

- ***shape transition, shape coexistence, shape mixing***
- ***drastic changes in structure with particle number, spin, excitation energy***
- ***"sudden" onset of deformation for N>58 neutron number***

Challenges for theory

- ***realistic effective Hamiltonians in adequate model spaces, beyond-mean-field methods***

aiming to

- ***unitary description of evolution in structure at low and high spins***
- ***comprehensive understanding of structure phenomena and β-decay properties***

complex VAMPIR model family

- **the model space is defined by a finite dimensional set of spherical single particle states**

- **the effective many-body Hamiltonian is represented as a sum of one- and two-body terms**

- **the basic building blocks are Hartree-Fock-Bogoliubov (HFB) vacua**

- **the HFB transformations are essentially *complex* and allow for proton-neutron, parity and angular momentum mixing being restricted by time-reversal and axial symmetry**

(T=1 and T=0 neutron-proton pairing correlations already included at the mean-field level)

- **the broken symmetries (s=N, Z, I, p) are restored by projection before variation**

**** The models allow to use rather large model spaces and realistic effective interactions***

Beyond-mean-field variational procedure: complex EXCITED VAMPIR model

Vampir

$$E^s[F_1^s] = \frac{\langle F_1^s|\hat{H}\hat{\Theta}_{00}^s|F_1^s\rangle}{\langle F_1^s|\hat{\Theta}_{00}^s|F_1^s\rangle}$$

Θ^s_{00} - symmetry projector

$| F^s_1 >$ - HFB vacuum

$$|\psi(F_1^s); sM\rangle = \frac{\Theta_{M0}^s|F_1^s\rangle}{\sqrt{\langle F_1^s|\hat{\Theta}_{00}^s|F_1^s\rangle}}$$

Excited Vampir

$$|\psi(F_i^s); sM\rangle = \Sigma_{j=1}^i |\phi(F_j^s)\rangle\, \alpha_j^i \qquad \text{for} \quad i = 1, ..., n-1$$

$$|\phi(F_i^s); sM\rangle = \Theta_{M0}^s|F_i^s\rangle$$

$$|\psi(F_n^s); sM\rangle = \Sigma_{j=1}^{n-1} |\phi(F_j^s)\rangle\, \alpha_j^n + |\phi(F_n^s)\rangle\, \alpha_n^n$$

$$(H - E^{(n)}N)f^n = 0$$

$$(f^{(n)})^+ N f^{(n)} = 1$$

$$|\Psi_\alpha^{(n)}; sM > = \sum_{i=1}^{n} |\psi_i; sM > f_{i\alpha}^{(n)}, \qquad \alpha = 1, \ldots, n$$

A~100 mass region

^{40}Ca - core

model space for both protons and neutrons :

$1p_{1/2}$ $1p_{3/2}$ $0f_{5/2}$ $0f_{7/2}$ $2s_{1/2}$ $1d_{3/2}$ $1d_{5/2}$ $0g_{7/2}$ $0g_{9/2}$ $0h_{11/2}$

(single-particle energies adjusted within complex MONSTER (VAMPIR))

renormalized G-matrix (OBEP, Bonn A/CD)

- ***pairing properties enhanced by short range Gaussians for:***

  T = 1 pp, np, nn channels

  T = 0, S = 0 and S = 1 channels

- ***onset of deformation influenced by monopole shifts:***

  $<0g_{9/2}\ 0f;\ T=0\ |G|\ 0g_{9/2}\ 0f;T=0>$

- ***Coulomb interaction between valence protons added***

Beta-decay formalism

Allowed and first-forbidden matrix elements		k	Δl	Δπ
Allowed				
$M(\rho_V, \lambda = 0)$	$C_V \int 1$	0	0	+1
$M(j_A, \kappa = 0, \lambda = 1)$	$C_A \int \vec{\sigma}$	1	0,±1 (no 0→0)	+1
First-forbidden				
$M(\rho_A, \lambda = 0)$	$C_A \int \gamma_5$	0	0	-1
$M(j_A, \kappa = 1, \lambda = 0)$	$C_A \int (\vec{\sigma} \cdot \vec{r}/i)$			
$M(\rho_V, \lambda = 1)$	$C_V \int \vec{r} i$	1	0,±1 (no 0→0)	-1
$M(j_V, \kappa = 0, \lambda = 1)$	$C_V \int \vec{\alpha}$			
$M(j_A, \kappa = 1, \lambda = 1)$	$C_A \int (\vec{\sigma} \times \vec{r})$			
$M(j_A, \kappa = 1, \lambda = 2)$	$C_A \int iB_{ij}$	2	0,±1, ±2 (no 0→0,1→0,0→1)	-1

	Reduced single-particle matrix elements in harmonic oscillator basis
"$\int \vec{\sigma}\cdot\vec{r}$"	$(-1)^{l_n-\frac{1}{2}+J}\delta_{j_n j_p}\begin{Bmatrix}1/2 & 1/2 & 1\\ l_n & l_p & j_n\end{Bmatrix}\sqrt{\frac{6(2l_n+1)(2j_p+1)}{4\pi}}\langle l_n 010\vert l_p 0\rangle\left(b\sqrt{n_n+l_n+3/2}\,\delta_{n_n,n_p}\delta_{l_p,l_n+1}-b\sqrt{n_n}\delta_{n_p,n_n-1}\delta_{l_p,l_n+1}\right)$
"$\int \vec{\sigma}\cdot\vec{\nabla}$"	$\begin{cases}(-1)^{l_n+\frac{1}{2}+j_n}\delta_{j_n j_p}\,\delta_{l_p\,l_n-1}\begin{Bmatrix}1/2 & 1/2 & 1\\ l_n & l_p & j_n\end{Bmatrix}\sqrt{6(2j_p+1)}\left(-\sqrt{l_p+1}\right)\frac{1}{b}\left(\sqrt{n_p+l_p+3/2}\,\delta_{n_n,n_p}-\sqrt{n_p}\delta_{n_p,n_n+1}\right)\\ (-1)^{l_n+\frac{1}{2}+j_n}\delta_{j_n j_p}\delta_{l_p\,l_n+1}\begin{Bmatrix}1/2 & 1/2 & 1\\ l_n & l_p & j_n\end{Bmatrix}\sqrt{6(2j_p+1)}\sqrt{l_p}\,\frac{1}{b}\left(\sqrt{n_p+l_p+1/2}\,\delta_{n_n,n_p}+\sqrt{n_p+1}\delta_{n_p,n_n-1}\right)\end{cases}$

Partial half-life of a beta transition

$$\frac{1}{t_{1/2}} = \frac{f}{K}$$

$$f = \int_1^{W_0} C(W)W(W^2-1)^{1/2}(W_0-W)^2 F(Z,W)dW,$$

Gamow-Teller $C(W) = B(GT)$

$$B_{if}(GT) = \frac{g_A^2}{2J_i+1}|M_{GT}|^2$$

$$M_{GT} = \sum_{ab} m_{GT}(ab)\langle \xi_f J_f || [c_a^\dagger \tilde{c}_b]_1 || \xi_i J_i \rangle$$

first forbidden $0^- \to 0^+$ $C(W) = k + kb/W$

$$O(0^-) = \sum_{ab} o^{(0)}(0^-)(ab)\langle \xi_f J_f || [c_a^\dagger \tilde{c}_b]_0 || \xi_i J_i \rangle$$

$$k = \zeta_0^2 + \frac{1}{9}w^2 \qquad kb = \frac{2}{3}\mu_1\gamma_1[-\zeta_0 w]$$

$$\zeta_0 = V + \tfrac{1}{3} w W_0,$$
$$V = v + \xi w'.$$

$$w = -g_A \sqrt{3} \langle a || r [\boldsymbol{C}_1 \times \boldsymbol{\sigma}]^{(0)} || b \rangle$$

$$w^{'} = -g_A \sqrt{3} \langle a || \frac{2}{3} r I(1,1,1,1,r) [\boldsymbol{C}_1 \times \boldsymbol{\sigma}]^{(0)} || b \rangle$$

$$v = \frac{\epsilon_{mec} g_A \sqrt{3}}{M} \langle a || [\boldsymbol{\sigma} \times \boldsymbol{\nabla}]^{(0)} || b \rangle$$

$$\boldsymbol{C}_{lm} = \sqrt{\frac{4\pi}{2l+1}} \boldsymbol{Y}_{lm}$$

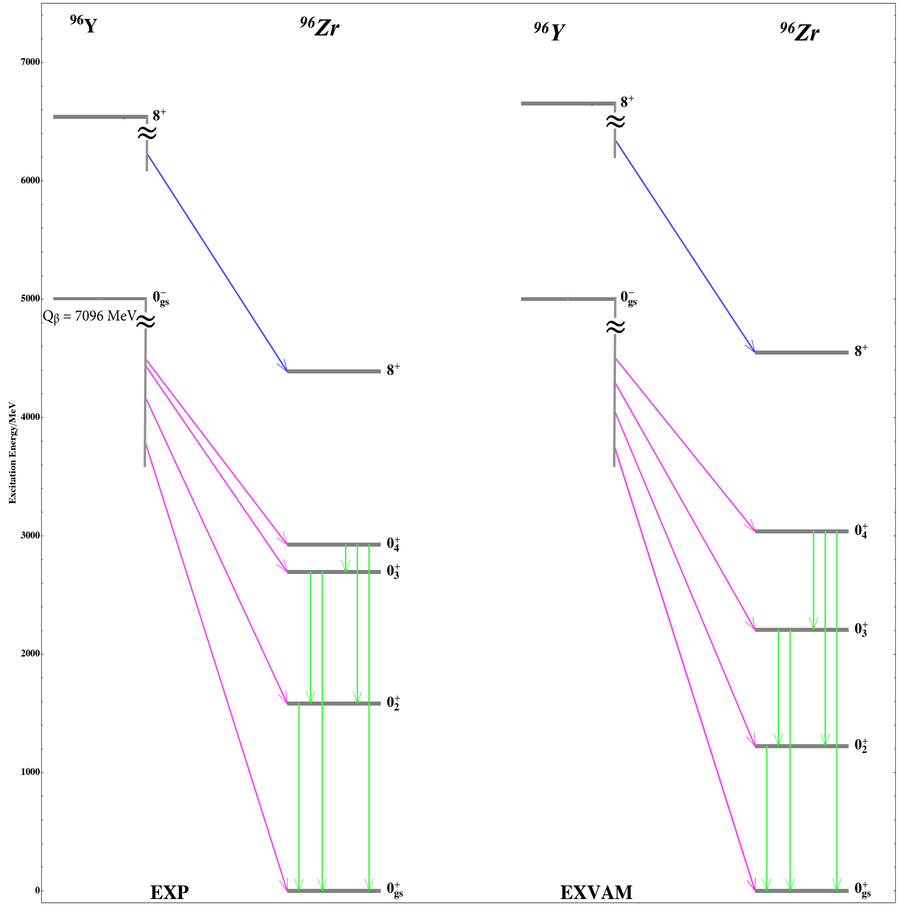
$$I(1,1,1,1,r) = \frac{3}{2} \begin{cases} 1 - \frac{1}{5}(\frac{r}{R})^2 & 0 \leq r \leq R \\ \frac{R}{r} - \frac{1}{5}(\frac{R}{r})^3 & r \geq R. \end{cases}$$

Triple shape coexistence in N=57 ^{96}Y and N=56 ^{96}Zr

A. Petrovici and A.S. Mare, Phys. Rev. C 101, 024307 (2020)

$E^{EXP}(8^+_{isomer})$=1.541 MeV

$E^{EXVAM}(8^+_{isomer})$=1.653 MeV



$E^{EXP}(8^+_{yrast})$ = 4.390 MeV

$E^{EXVAM}(8^+_{yrast})$ = 4.549 MeV

^{96}**Y :** - ***spherical*** *configuration in* 0^-_{gs}

- ***prolate*** *mixing in* 8^+_{isomer}

(Q^{EXP}_{sp} = -98(11) efm^2 Q^{EXVAM}_{sp} = -97.3 efm^2)

^{96}**Zr :** - ***triple shape coexistence*** *in lowest four* 0^+ *states*

- ***prolate-oblate coexistence*** *in* 8^+ *daughter states*

Triple shape coexistence and E0 transitions in ^{96}Zr

The amount of mixing for the lowest 0^+ states of ^{96}Zr.

$I[\hbar]$	spherical	prolate	oblate
0_1^+	97%	1%	2%
0_2^+	19%	52%	29%
0_3^+	30%	50%	20%
0_4^+	45%	3%	52%

$\rho^2(E0)$ values for the lowest four 0^+ states in ^{96}Zr .

$I[\hbar]$	EXVAM 0_2^+	0_3^+	0_4^+	Exp. 0_2^+
0_1^+	0.021	0.005	0.008	0.0075(14)
0_2^+		0.053	0.010	
0_3^+			0.016	

Exp: $\rho^2_{32}/\rho^2_{31} = 9.4(26)$; $\rho^2_{42}/\rho^2_{41} < 3.0$

$I[\hbar]$	$\epsilon_{\rm mec} = 1.15$	$\epsilon_{\rm mec} = 1.19$	Exp.
0^+_{gs}	5.60	5.53	5.59 (1)
0^+_2	6.40	6.32	6.97 (4)
0^+_3	6.52	6.45	7.41 (6)
0^+_4	6.48	6.42	7.92 (9)

$T^{EXVAM}_{1/2}$ (0^- (ground state)) = 5.21 **s** ($\epsilon_{\rm mec} = 1.15$)

$T^{exp}_{1/2}$ (0^-(ground state)) = 5.34 (5) **s**

Independent chains of variational calculations for the parent and daughter states

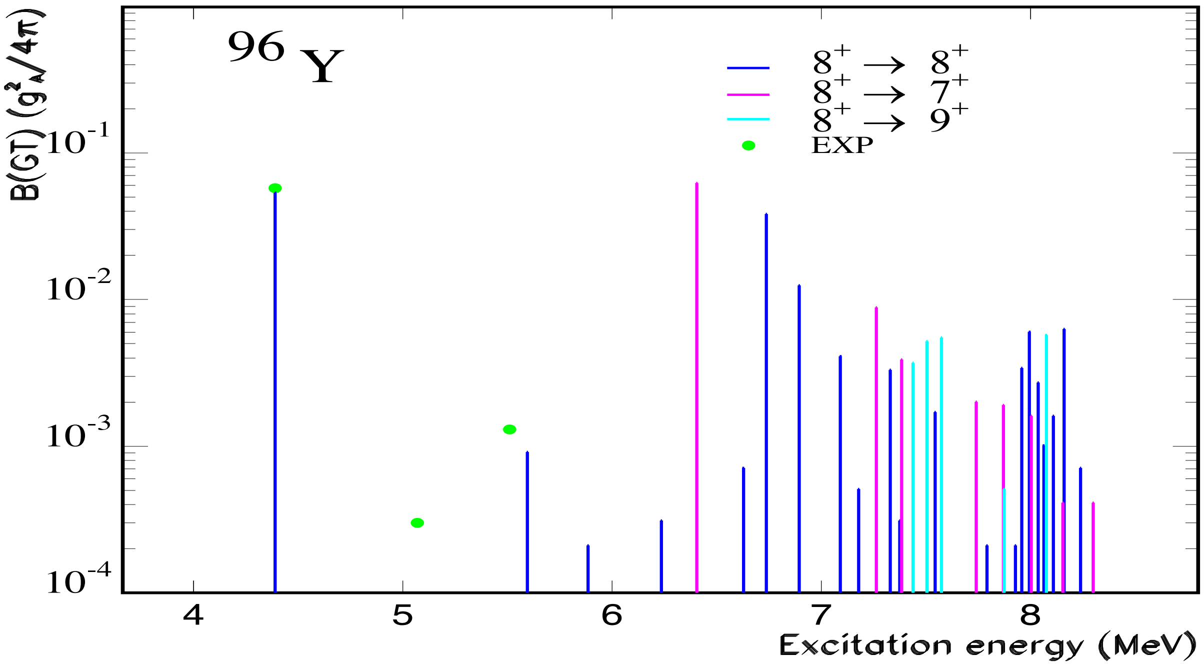
^{96}Y : *mixing of prolate deformed configurations for* 8^+_{isomer}

^{96}Zr : *prolate-oblate coexistence and variable mixing for daughter* $8^+, 7^+, 9^+$ *states*

Gamow-Teller transition probabilities

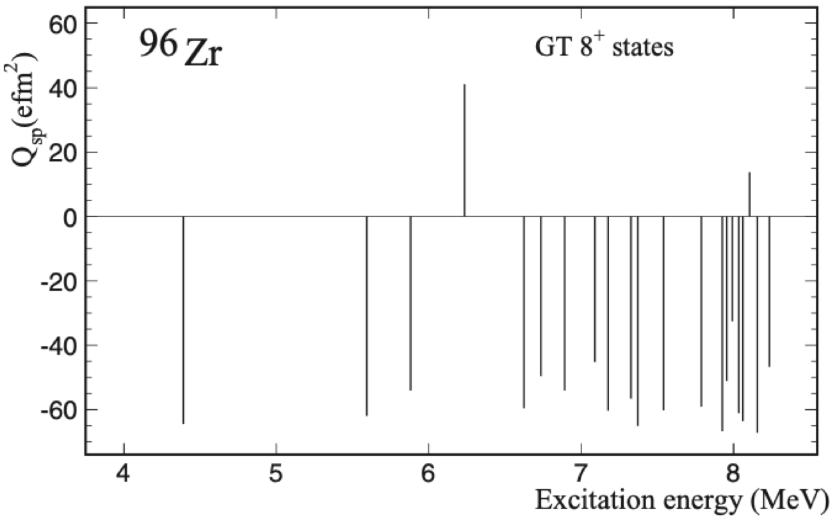
$$B_{if}(GT) = \frac{1}{2J_i+1}\frac{g_A^2}{4\pi}|M_{GT}|^2$$

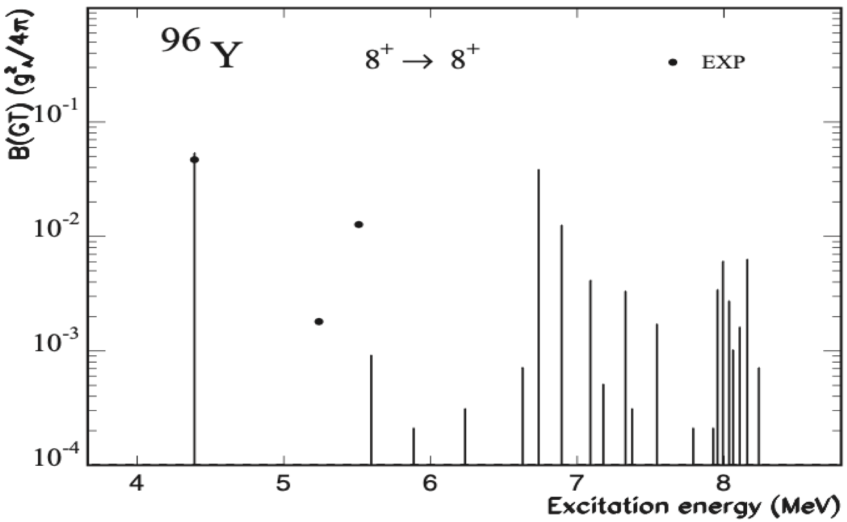
$$M_{GT} \equiv (\xi_f J_f||\hat{\sigma}||\xi_i J_i)$$
$$= \sum_{ab} M_{GT}(ab)(\xi_f J_f||[c_a^\dagger \tilde{c}_b]_1||\xi_i J_i)$$

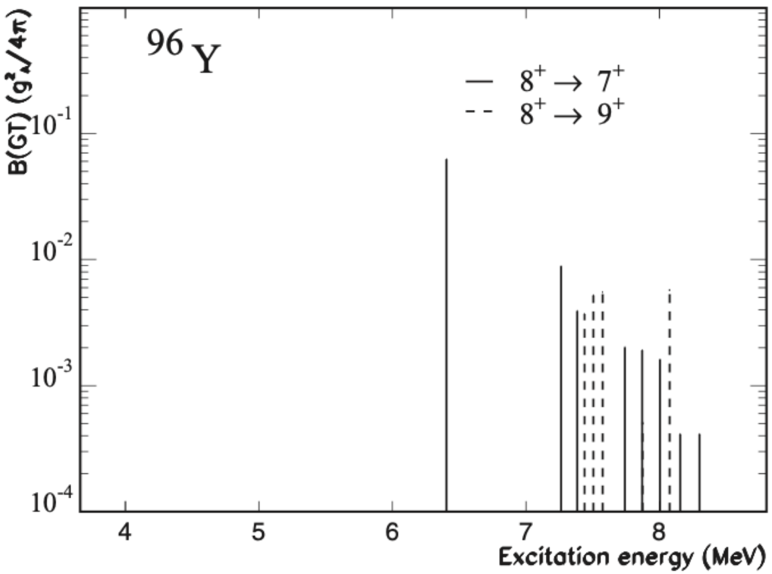


$T^{exp}_{1/2}(8^+(1.541\ MeV)) =$ **9.6 s** $\quad T^{EXVAM}_{1/2}(8^+(1.653\ MeV)) =$ **9.3 s**

$Q_\beta = 7.096$ MeV







$E^{EXVAM}(8^+_{yrast}) = 4.549$ MeV

$E^{EXP}(8^+_{yrast}) = 4.390$ MeV

Summary and outlook

complex* EXCITED VAMPIR model *self-consistently describes

- ***experimental trends in the N=58 Kr, Sr, and Zr isotopes:***
  - ***^{96}Sr and ^{98}Zr***

    - ***triple shape coexistence***: **spherical**, **prolate**, and **oblate** configurations **mixed** in the lowest 4 **0^+** states

    - ***multifaceted structure*** with increasing spin and energy corroborated with electromagnetic properties
  - ***^{94}Kr***

    - *evolution of* ***oblate-prolate mixing*** with increasing spin and excitation energy

- ***particular behavior of the N=56 ^{100}Ru isotope***

- ***triple shape coexistence effects on structure and β^- decay of the N=57 ^{96}Y to N=56 ^{96}Zr:***

  - ***^{96}Y***

    - ***triple shape coexistence***: - **spherical 0^-** ground state **– first-forbidden β^- decay**
      - **oblate** low-lying states; **prolate 6^-** isomer
      - **mixing** of **prolate** configurations in **8^+_{isomer} – GT β^- decay**
  - ***^{96}Zr***
    - ***triple shape coexistence***: - **spherical**, **prolate**, and **oblate** configurations **mixed** in the lowest 4 **0^+** states *(significant **E0** strengths)*
      - ***prolate-oblate mixing*** *in the structure of* ***8^+, 7^+, 9^+* – GT daughter** states